1992

1a. $f' = 15x^4 - 15x^2$

$= 15x^2(x^2-1)$

$= 15x^2(x+1)(x-1)$

$x=0 \quad x=-1 \quad x=1$

		-1		0		1	
x^2	+		+	0	+		+
$x+1$	−	0	+		+		+
$x-1$	−		−		−	0	+
f'	+	0	−	0	+	0	+

Inc $(-\infty,-1) \cup (1,\infty)$

b. $f'' = 60x^3 - 30x$

$= 30x(2x^2-1)$

$30x=0 \qquad 2x^2-1=0$

$x=0 \qquad 2x^2=1$

$x = \pm\sqrt{\tfrac{1}{2}}$

		$-\sqrt{\frac{1}{2}}$		0		$\sqrt{\frac{1}{2}}$	
$30x$	——		——	0			
$x-\sqrt{\frac{1}{2}}$	——		——		——	0	
$x+\sqrt{\frac{1}{2}}$	−	0					
	−	0	+	0	−	0	+

UP $(-\sqrt{\tfrac{1}{2}},0)$ $(\sqrt{\tfrac{1}{2}},\infty)$

c. Eqn of horiz. tangents, where $m=0$

x	$f(x)$
0	$3(0)^5-5(0)^3+2=2$
−1	4
1	0

Eqns. $y=2$

$y=4$

$y=0$

2a. $v = 3(t-1)(t-3) = 3(t^2-4t+3)$

$= 3t^2-12t+9$

$a(t) = \frac{dv}{dt} = 6t-12$

Take the first deriv of accel to find the ~~a~~ min.

$a' = 6$ ∴ always positive

No critical pts of a', must use end pts 0 & 5

0	$6(0)-12=-12$ MIN
5	$6(5)-12=18$

MIN ACCEL $= -12$ units/sec^2

b. Total distance

$v(t) = 3(t-1)(t-3)$

	0		1		3		5
$t-1$		−	0				
$t-3$		−	−	−	0		
		+	0	−	0	+	

Total = (0 to 1) + (1 to 3) + (3 to 5)

Must find distance formula

$\text{dist} = \int 3t^2-12t+9$

$= t^3-6t^2+9t+c$

$\text{dist}(2)=0$

$0 = 2^3-6(2)^2+9(2)+c$

$c=-2$

$\text{dist} = t^3-6t^2+9t-2$

x	$f(x)$
0	$0^3-6(0)^2+9(0)-2=-2$
1	2
3	−2
5	18

0 to 1 = 4

1 to 3 = 4

3 to 5 = 20

28 units

1992

2c
$$\text{Avg Velocity} = \frac{\int_0^5 \text{velocity}}{5-0}$$

$$\text{Avg} = \frac{\int_0^5 3t^2 - 12t + 9}{5-0} = 4 \text{ units/sec}$$

or

$$\text{Avg Velocity} = \frac{\text{dist}(5) - \text{dist}(0)}{5-0} = 4$$

#3 a. $\mathbb{R}$ $x \neq 0$ $\left|\frac{x}{1+x^2}\right|$ must be positive

b. $\ln\left|\frac{-x}{1+x^2}\right| = \ln\left|\frac{x}{1+x^2}\right|$

if $f(x) = f(-x)$ then EVEN

c.

$$f' = \frac{1}{\frac{x}{1+x^2}} \cdot \frac{(1+x^2)(1) - x(2x)}{(1+x^2)^2}$$

$$= \frac{\cancel{1+x^2}}{x} \cdot \frac{1+x^2 - 2x^2}{(1+x^2)^{\cancel{2}}}$$

$$= \frac{1-x^2}{x(1+x^2)}$$

Critical pts $\cancel{0}$, 1, −1

undefined

	−1		0		1	
$1-x$	+	+	+	+	0	−
$1+x$	−	0	+	+	+	+
x	−	−	0	+	+	+
f'	+	0 −	⊗	+	0	−

max at $x=-1$ max at $x=1$

d. $f(x) \le \ln\left(\frac{1}{2}\right)$

4a. $y + \cos y = x + 1$

$$\frac{dy}{dx} - \sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(1 - \sin y) = 1$$

$$\frac{dy}{dx} = \frac{1}{1-\sin y}$$

b. Vert tangent where slope is undefined

$$1 - \sin y = 0$$
$$-\sin y = -1$$
$$\sin y = 1$$
$$\sin^{-1}(1) = \frac{\pi}{2}$$

Find the corresponding x coordinate

$$y + \cos y = x + 1$$
$$\frac{\pi}{2} + \cos\frac{\pi}{2} = x + 1$$
$$\frac{\pi}{2} + 0 = x + 1$$
$$\boxed{\frac{\pi}{2} - 1 = x}$$

c. $$\frac{dy}{dx} = \frac{1}{1-\sin y} = (1-\sin y)^{-1}$$

$$\frac{d^2y}{dx^2} = -(1-\sin y)^{-2}\left(-\cos y \frac{dy}{dx}\right)$$

substitute from part 'a'

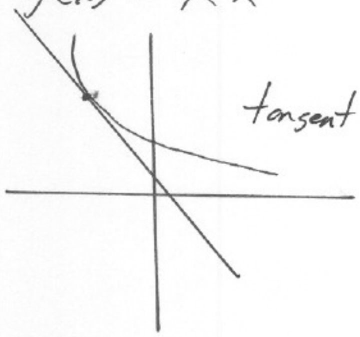
$$\frac{d^2y}{dx^2} = \frac{-1}{(1-\sin y)^2}\left(-\cos y \cdot \frac{1}{1-\sin y}\right)$$

$$= \frac{\cos y}{(1-\sin y)^3}$$

1992

#5 a. $f(x) = e^{-x}$

$g(x) = kx$



tangent at a point

$m f(x) = f'(x) = -e^{-x}$

$m g(x) = g'(x) = k$

$k = -e^{-x}$

$\cancel{k}$

At the point where they touch the eqns must be equal to each other $\therefore$ $e^{-x} = kx$

solve for x & k

$k = -e^{-x}$

$e^{-x} = kx$

$e^{-x} = -e^{-x} x$

$\frac{e^{-x}}{-e^{-x}} = x$

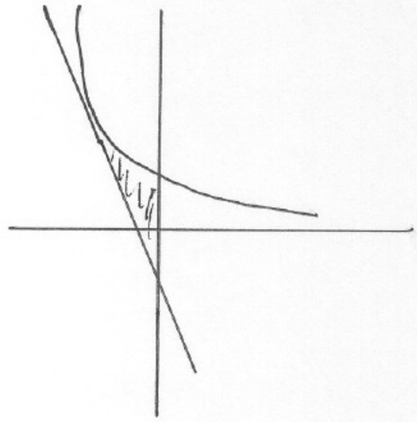
$-1 = x$

from before

$k = -e^{-x}$

$k = -e^{-(-1)} = -e^{1}$

b.



$\int_{-1}^{0} \left[e^{-x} - (-ex)\right] dx$

$= \int_{-1}^{0} \left[e^{-x} + ex\right] dx =$

$-e^{-x} + \frac{ex^2}{2} \Big]_{-1}^{0}$

$= \frac{e}{2} - 1$

c. $\pi \int_{-1}^{0} (e^{-x})^2 - (-ex)^2 \, dx$

1992 #6

a $\frac{dv}{dt} = \frac{k}{r}$ $t=0$ $r=1$

$t=15$ $r=2$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4\pi r^2 \frac{dr}{dt} = \frac{k}{r}$

$\frac{dr}{dt} = \frac{k}{r} \cdot \frac{1}{4\pi r^2} = \frac{k}{4\pi r^3}$

$4\pi r^3 dr = k\,dt$

$\int 4\pi r^3 dr = \int k\,dt$

$\frac{4\pi r^4}{4} = kt + c$

$\pi r^4 = kt + c$

at $t=0$ $r=1$

$\pi(1)^4 = k(0) + c$

$c = \pi$

$\therefore \pi r^4 = kt + \pi$

at $t=15$ $r=2$

$\pi 2^4 = k(15) + \pi$

$16\pi = 15k + \pi$

$15\pi = 15k$

$k = \pi$

$\therefore \pi r^4 = \pi t + \pi$

$r^4 = \frac{\pi t + \pi}{\pi} = t + 1$

$r = \sqrt[4]{t+1}$

b. $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\sqrt[4]{t+1}\right)^3$

$V(0) = \frac{4}{3}\pi\left(\sqrt[4]{0+1}\right)^3 = \frac{4}{3}\pi$

Volume needs to be 27 times this amount

$V = 27 \cdot \left(\frac{4}{3}\pi\right) = 36\pi$

$36\pi = \frac{4}{3}\pi\left(\sqrt[4]{t+1}\right)^3$

$27 = (t+1)^{\frac{3}{4}}$

$27^{\frac{4}{3}} = t+1$

$81 = t+1$

$80 = t$